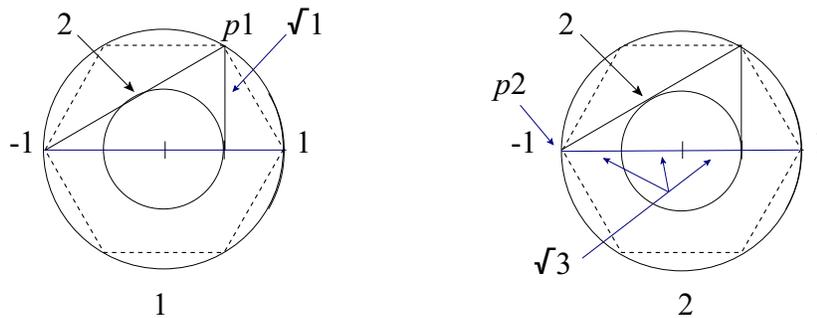


## Geometric Interpretation of Hardy's Paradox

(Jack's Lecture at the University)

For Hardy's paradox, the graphic structure below calculates the angular position in rotation through the waveform to its linear components using the cosine function. The angle subtended to the events [Both In] and [Both Out] is  $P(\theta) = \cos^2 \theta$ . This gives the quantum-level probability for the set of two paths in the positions ( $p1$ ) and ( $p2$ ). When the values generated are assessed within the geometry, they explain the discrepancy between the pure values in Aharonov's calculations<sup>1</sup> and the experiment conducted by Lundeen and Steinberg, *Experimental joint weak measurement on a photon pair as a probe of Hardy's Paradox*<sup>2</sup>



The positions  $p1$  and  $p2$  are rotations through the simultaneous firing of both ports D

[Both In]	[Both Out]
(Inner paths)	(Outer paths)
$(\sqrt{1})/2 = 0.50$	$\sqrt{3}/2 = 0.87$
$(0.50)^2 = 0.25$	$(0.87)^2 = 0.75$
$\cos^2 (60) = 0.25$	$\cos^2 (30) = 0.75$

**Figures 1, 2.** In Hardy's paradox, the experimental apparatus allows post-selection by weak measurement of two parallel quantum states. Two entangled photons traverse the apparatus across four possible paths and exit at two ports, each with two orthogonal paths of exit. The entanglement of the photons is analysed for the Dark Ports that simultaneously receive the photons, [Both In] at ( $p1$ ) and [Both Out] at ( $p2$ ).

### Calculation of the Trigonometric Values

In the geometric interpretation, numerical counting and linear measurement are equal parameters for calculation of the resulting values. For the option of counting values, the segments across the structure incorporate dimensional boundaries. Vectors that eccentrically cross a dimensional boundary have the square root assigned. The two segments of the hypotenuse cancel the square root by multiplication as their endpoints are on the same dimensional level, and the value given is 2. The value for Both Out (0.75) projects from the negative side of the x-axis and takes a negative value in Table 3. The geometric model is a composite geometry developed dynamically and sequentially from a *sub-classical, primordial null state* across identifiable dimensional boundaries in a process best described as *self-organization* and *least action*.

**The Tables - Hardy's Paradox**

**Table 1: Theoretical Weak Values from the Aharonov Paper**

I = inner path, O = outer path

	N(I-)	N(O-)	
N(I+)	<b>0</b>	1	1
N(O+)	1	<b>-1</b>	0
	1	0	

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**Table 2: Experimental Weak Values**

	N(I-)	N(O-)	
N(I+)	<b>0.245</b>	0.641	0.886
N(O+)	0.719	<b>-0.759</b>	-0.04
	0.964	-0.118	

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**Table 3: Application of the Cosine Function to the Data Based on the Geometric Model**

Table 3 defines the non-collapsed wave-form as a function of angular rotation in the geometry. The two calculations represent the contribution when Dark Ports fire simultaneously.

	N(I-)	N(O-)	
N(I+)	<b><math>\cos^2(60) = 0.25</math></b>	Not Applied	---
N(O+)	Not Applied	<b><math>-\cos^2(30) = -0.75</math></b>	---
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**Summary:** The agreement between Table 2 and Table 3 validates the geometric interpretation by applying the cosine function to the data. The difference between the theoretical values in Table 1 and the experimental results in Table 2 arises because the theoretical data does not account for the geometric structure, which incorporates the dynamic mechanism of paradox across dimensional boundaries.

Finally, a unique feature of Hardy's paradox is that the geometric representation demonstrates a dualism of equivalence between linear measurement and counting across dimensional boundaries for the cosine squared function.

**Perspective:** The mathematical basis of Hardy's paradox is explained through quantum formalism, which incorporates the imaginary number  $i, \sqrt{-1}$ . The term imaginary is used because, in classical mathematics, the antecedents,  $(+1)^2$  and  $(-1)^2$  should both produce the product  $+1$ . The reverse operation of taking the square root is then only in the form  $(\sqrt{+1})$  and not  $\sqrt{-1}$  as in quantum structure. Thus, the classical antecedents have no meaning in quantum format. They are imaginary in quantum formalism the same way that  $\sqrt{-1}$  is imaginary in classical formalism.

This is where a fundamental dualism arises both in the transformation from quantum to classical mathematical structure, and within each for a component that has no meaning. A dimensional boundary exists across which the only available mechanism of relationship is a *collapse*. In physics, this is referred to as the collapse of the wavefunction.

What is meant by the term *classical level*? The fact that the square root function must be applied to the value -1 to produce a quantum format means that the dimensional basis of classical structure is higher than its dimensionally simpler quantum parent. Consider that only the x-axis of quantum, two-dimensional structure is *real*. The y-axis has  $\sqrt{-1}$  attached. Thus, there is an underlying paradoxical dualism across quantum and classical structures. Since paradox is prohibited in formal mathematics the dualism that arises between these two formats is irrevocable for the creation of a single basis across them. It is also a clue to the role of paradox in the many ways we consider universal structures.

At the classical level the universal presence of underlying paradox for both logic and mathematics is handled by the creation of a dualism in which each half is consistent in its own realm but not to the other half. This gives rise to the conundrum that we find paradox is a natural feature of relationship across structures that still have common, identified properties. Each example will be unique, but the form is the same. Two elements are contained in a larger common structure of relationship that does not distinguish between the parts for their defining property. However, when we examine each part separately they have nothing in common. This is a paradox in their relationship.

Paradox takes two forms, static and dynamic. The static form is as described above. For dynamic structure, a fundamental process of outward cycle occurs in an attempt to normalize containment across paradoxical elements. Complexity is subsumed as the structure grows in additional cycles. In both forms, static and dynamic, normalization is not possible, and the state remains incomplete.

In a nutshell, classical and quantum structures are inconsistent bases to each other across a dimensional boundary. And each will necessarily be incomplete for description of the universe. The native universe cannot be described in a consistent framework because of the role of paradox. From its initial, null form, the native universe develops dynamically across absolute and paradoxical dimensional boundaries. Complexity develops and is sequentially subsumed as the state grows its structure.

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<sup>1</sup> Y. Aharonov et al. Phys. Lett. A 301, 130 (2002).

<sup>2</sup> <https://arxiv.org/abs/0810.4229>